

# IN SEARCH OF THE NEXT NEXT-TO-MSSM <sup>1</sup>

S. A. Abel

Service de Physique Théorique

Université Libre de Bruxelles

Boulevard de Triomphe

Bruxelles 1050, Belgium

## Abstract

It is argued that the Next-to-MSSM (NMSSM) is unnatural from the point of view of cosmology and fine tuning. In particular, such singlet extensions to the MSSM do not provide a simple solution to the ‘ $\mu$ -problem’. Models with singlets can be constructed using gauged- $R$  symmetry or target space duality. However their superpotentials have terms in addition to those of the NMSSM.

---

<sup>1</sup>Talk presented at , “Electroweak Interactions and Unified Theories”, XXXIst Rencontres de Moriond, March 16-23, 1996

# 1 Introduction

According to current theoretical prejudice, the Minimal Supersymmetric Standard Model (MSSM) is the most likely candidate for physics beyond the standard model [1]. One of its most interesting features is an upper bound on the mass of the lightest higgs boson of about 120 GeV or so. This comes about because of the rather restricted form of the superpotential;

$$W_{MSSM} = h_u Q_L H_2 U_R^c + h_d Q_L H_1 D_R^c + h_e L H_1 E_R^c + \mu H_1 H_2, \quad (1)$$

where  $Q_L$ ,  $L$ ,  $H_1$ , and  $H_2$  are the standard doublet superfields. Because of this, and also because of the fine-tuning problem inherent in the (phenomenologically necessary) choice of  $\mu \sim M_W$  (sometimes referred to as the  $\mu$ -problem [2, 3]), many authors instead consider a variant of this superpotential known variously as the Minimally-Extended-MSSM or Next-to-MSSM (NMSSM) [4];

$$W_{NMSSM} = h_u Q_L H_2 U_R^c + h_d Q_L H_1 D_R^c + h_e L H_1 E_R^c + \lambda N H_1 H_2 - \frac{k}{3} N^3, \quad (2)$$

where  $N$  is an additional singlet superfield. In these models the  $\mu$ -term is generated radiatively when the latter acquires a vacuum expectation value on the breaking of electroweak symmetry. Since  $\langle n \rangle \sim M_W$ , this was thought to be a potential simple solution to the  $\mu$ -problem.

Because of difficulties with cosmology (specifically the appearance of domain walls) this appears to be no longer the case [5, 6], however in view of the less constrained higgs phenomenology, it is still worth pursuing models with singlet extensions. Here I wish to argue that the most natural way to avoid cosmological problems in models with extra singlets is to introduce  $\mu$  terms *in addition* to the terms in eq.(2). I shall show that this may be done by borrowing from another solution to the  $\mu$ -problem, proposed by Giudice and Masiero. Thus, if one wishes to consider singlet extensions of the MSSM, it is more natural to consider the most general form of the superpotential [7],

$$W_{N^2MSSM} = h_u Q_L H_2 U_R^c + h_d Q_L H_1 D_R^c + h_e L H_1 E_R^c + \mu H_1 H_2 + \mu' N^2 + \lambda N H_1 H_2 - \frac{k}{3} N^3. \quad (3)$$

The second point I shall discuss, is the fact that the singlet field,  $N$ , is a potential hazard to the gauge-hierarchy. This is because singlet fields can give rise to divergent tadpole diagrams. Clearly, at the level of global supersymmetry, there is no gauge-symmetry one can give to  $N$  which forbids such operators (that is linear  $N$  terms) appearing. In fact there is a large number of potentially dangerous operators which must be set to zero by hand unaided by any symmetry. At the level of supergravity however, two suitable symmetries become available. These are gauged  $R$ -symmetry, and target-space duality symmetry. In both these cases it is

possible to construct models which have no cosmological problems, no fine-tuning problems and no arbitrarily forbidden operators.

## 2 Cosmological Problems in the NMSSM

First let us discuss the cosmological problems facing the simple NMSSM. In addition to the terms derived from the superpotential in eq.(2), there are soft-supersymmetry breaking terms. It is these which are responsible for the breaking of electroweak symmetry, and in this case, terms of particular importance are the trilinear scalar couplings ( $A$ -terms) which appear in the potential. Together with the supersymmetric contribution, they lead to a scalar potential of the form

$$\begin{aligned}
 V_{soft} &= -\lambda A_\lambda n h_1 h_2 - \frac{k}{3} A_k n^3 - \lambda k h_1 h_2 n^{*2} + \text{h.c.} + \dots \\
 &= -2(\lambda A_\lambda \langle |n| \rangle \langle |h_1^0| \rangle \langle |h_2^0| \rangle \cos(\theta_1 + \theta_2 + \theta_n) - \frac{k}{3} A_k \langle |n| \rangle^3 \cos(3\theta_n) \\
 &\quad - \lambda k \langle |h_1^0| \rangle \langle |h_2^0| \rangle \langle |n| \rangle^2 \cos(\theta_1 + \theta_2 - 2\theta_n)) + \dots
 \end{aligned} \tag{4}$$

where small letters indicate scalar components of superfields, and where for convenience I have taken  $\lambda$ ,  $k$ ,  $A_\lambda$  and  $A_k$  to be real. The dots above stand for terms which either do not get a VEV on electroweak symmetry breaking, or are independent of the phases of the higgs scalars ( $\theta_1$ ,  $\theta_2$  and  $\theta_n$ ). For suitable choices of the parameters (e.g. all positive) the above potential is clearly minimised where all the cosines are +1 which has three solutions, namely  $\theta_i = 0$  or  $\pm 2\pi/3$ . Field configurations which interpolate between any two minima are topologically stable and lead to domain walls. Solving for these is a relatively straightforward matter, and this was done in ref.[6] where, not surprisingly (given that all the VEVs and  $A_\lambda$  and  $A_k$  are  $\sim M_W$ ), it was found that the walls have mass per unit area

$$\sigma \sim M_W^3 \sim 10^5 \text{ kg cm}^{-2}. \tag{5}$$

Such walls are a cosmological disaster since, for example, their density falls as  $T^2$  whereas that of radiation falls as  $T^4$  so they eventually dominate and cause power law inflation [6].

There are a number of solutions which one could consider to rectify this situation. One which I shall not discuss in much detail here is to embed the discrete symmetry in a broken gauge symmetry [8]. In this case the degenerate vacua are connected by a gauge transformation in the full theory [8, 9]. After the electroweak phase transition, one expects a network of domain walls bounded by cosmic strings to form and then collapse. This situation was examined in ref.[9], where the conclusion was that rather complicated cosmological scenarios are required in

order to be able to accommodate it. The most natural solution, which is to simply insist that the  $Z_3$  symmetry be explicitly broken, will be the main focus in what follows.

### 3 Breaking $Z_3$

In principle, the  $Z_3$  symmetry need not be broken by very much in order to solve the domain wall problem. This was pointed out by Zel'dovich *et al* albeit in a rather different context [10], and for the case at hand, it turns out that even gravitationally suppressed terms are sufficient to remove the walls before the onset of primordial nucleosynthesis (at  $t \sim 1$  sec) [6, 11]. (The release of entropy from walls collapsing after this time, would effect the primordial abundances.) For example if one adds a piece

$$W_\epsilon = \lambda' \frac{N^4}{M_{\text{Pl}}} \quad (6)$$

to the NMSSM superpotential, one requires only that  $\lambda' \gtrsim 10^{-7}$  in order to satisfy the above constraint. This is because the walls continually straighten under their own tension, with the typical radius of curvature increasing as  $t$ . Eventually even this tiny pressure comes to dominate over the tension.

In ref.[6] however, it was pointed out that this solution cannot work for the NMSSM. This is because all the operators suppressed by one power of  $M_{\text{Pl}}$  which one can write down, lead to a divergent two or three loop diagram of the form discussed in ref.[12]. Such diagrams lead to a term linear in  $n$ ;

$$\delta V \sim \frac{\lambda'}{16\pi^2} (n + n^*) M_{\text{Pl}} M_W^2. \quad (7)$$

This destabilises the Planck/weak hierarchy unless  $\lambda' \lesssim 10^{-11}$ , but such a small value is clearly in conflict with the previous constraint coming from nucleosynthesis. (By simple power counting one finds that this is the leading divergence which can occur.) These divergences can be calculated in the framework of  $N = 1$  supergravity [13], in which the model depends only of the Kähler function

$$\mathcal{G} = K(\Phi, \bar{\Phi}) + \log |\tilde{W}(\Phi)|^2. \quad (8)$$

The function  $K = \bar{K}$  is responsible for the kinetic terms, and the superpotential  $\tilde{W}$ , is a holomorphic function of the superfields (generically denoted by  $\Phi$  above). However, as we shall see, the effective low energy superpotential  $W$  may receive terms from  $K$  as well as  $\tilde{W}$ .

So any suitable  $Z_3$ -breaking model must have  $\mu$  or  $\mu' \neq 0$  in the effective low energy lagrangian. In addition, any solution which can achieve this must of course also ensure the absence of the  $N^4/M_{\text{Pl}}$  operator above. In fact a brief examination of the possible divergences, shows

that the operators  $NH_iH_i^\dagger$  and  $NNN^\dagger$  must be forbidden in  $K$ , and the following operators must be forbidden in  $\tilde{W}$ ;

Operator	Loop-order of divergent diagram
$N^2, H_1H_2$	1
$N^4, N^2H_1H_2$	2
$(H_1H_2)^2$	3
$N^2(H_1H_2)^3, N^4(H_1H_2)^2, N^6(H_1H_2), N^8$	5
$N^2(H_1H_2)^4, N^4(H_1H_2)^3, N^6(H_1H_2)^2, N^8(H_1H_2), N^{10}$	6

In particular, the presence of the  $N^2$  and  $H_1H_2$  operators in this list means that the  $\mu$  or  $\mu'$  terms must come from  $K$ . To achieve this without destabilising the hierarchy, one can simply add  $H_1H_2$  and/or  $N^2$  into  $K$  as was first suggested in ref.[3]. This indeed generates the desired  $\mu$  and/or  $\mu' \sim M_W$  terms in the effective low energy (global supersymmetry) theory.

However, an explanation for the absence of all the operators above, requires an additional symmetry, under which  $K$  and  $\tilde{W}$  transform differently. Two obvious examples are duality symmetry and gauged- $R$  symmetry [14]. To conclude I shall present an example of the latter [9].

In this case  $K$  has zero  $R$ -charge, but  $\tilde{W}$  has  $R$ -charge 2. This means that the standard renormalisable NMSSM higgs superpotential,

$$W_{\text{higgs}} = \lambda N H_1 H_2 - \frac{k}{3} N^3, \quad (9)$$

has the correct  $R$ -charge if  $R(N) = 2/3$  and  $R(H_1) + R(H_2) = 4/3$ . So consider the Kähler function

$$\mathcal{G} = z^i z_i^\dagger + \Phi \Phi^\dagger + \Phi' \Phi'^\dagger + \left( \frac{\alpha}{M_{\text{Pl}}^2} \Phi^\dagger \Phi' H_1 H_2 + \frac{\alpha'}{M_{\text{Pl}}^2} \Phi \Phi'^\dagger N^2 + \text{h.c.} \right) + \log |h(z) + g(\Phi, \Phi')|^2, \quad (10)$$

where  $h(z)$  is the superpotential involving just visible sector fields and  $\Phi, \Phi'$  here represent hidden sector fields with superpotential  $g(\Phi, \Phi')$  (they may represent arbitrary functions of hidden sector fields in what follows). Both  $\Phi$  and  $\Phi'^\dagger$  appear here in order to prevent unwanted couplings being allowed in the superpotential which must be a holomorphic function of superfields.

The invariance of  $K$  requires that  $R(\Phi) + R(\Phi'^\dagger) = 4/3$ . If, for example, one chooses the  $R$ -charges to be  $R(\Phi) = 16/3, R(\Phi') = 4$ , *all* of the dangerous operators are forbidden [9]. The form of the low energy superpotential is then that in eq.(3). This, more general singlet extension of the MSSM, therefore appears to be a much more natural choice from the point of view of cosmology and fine-tuning.

## References

- [1] For reviews see, H. P. Nilles, Phys. Rep. 110 (1984) 1; H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75.
- [2] L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1983) 2359; J. E. Kim and H. P. Nilles, Phys. Lett. B138 (1984) 150; K. Inoue, A. Kakuto and T. Takano, Prog. Theor. Phys. 75 (1986) 664; A. A. Ansel'm and A. A. Johansen, Phys. Lett. B200 (1988) 331.
- [3] G. Giudice and A. Masiero, Phys. Lett. B206 (1988) 480
- [4] P. Fayet, Nucl. Phys. B90 (1975) 104; H.-P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B120 (1983) 346; J.-M. Frère, D. R. T. Jones and S. Raby, Nucl. Phys. B222 (1983) 11; J.-P. Derendinger and C. A. Savoy, Nucl. Phys. B237 (1984) 307; B. R. Greene and P. J. Miron, Phys. Lett. B168 (1986) 226; J. Ellis, K. Enqvist, D. V. Nanopoulos, K. Olive, M. Quiros and F. Zwirner, Phys. Lett. B176 (1986) 403; L. Durand and J. L. Lopez, Phys. Lett. B217 (1989) 463; M. Drees, Intern. J. Mod. Phys. A4 (1989) 3645; J. Ellis, J. Gunion, H. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D39 (1989) 844; T. Elliott, S. F. King and P. L. White, Phys. Lett. B351 (1995) 213; S. F. King and P. L. White, Phys. Rev. D52 (1995) 4183.
- [5] S. A. Abel and P. L. White, Phys. Rev. D52 (1995) 4371.
- [6] S. A. Abel, S. Sarkar and P. L. White, Nucl. Phys. B454 (1995) 663.
- [7] A. T. Davies, C. D. Froggatt and R. G. Moorhouse, hep-ph/9603388
- [8] G. Lazarides and Q. Shafi, Phys. Lett. B115 (1982) 21; T. W. B. Kibble, G. Lazarides and Q. Shafi, Phys. Rev. D26 (1982) 435; Phys. Lett. B113 (1982) 237; S. M. Barr, D. B. Reiss and A. Zee, Phys. Lett. B116 (1982) 227.
- [9] S. A. Abel, ULB-TH-96/03, hep-ph/9603301
- [10] Ya. B. Zel'dovich, I. Yu. Kobzarev and L. B. Okun, Sov. Phys. JETP 40 (1975) 1.
- [11] J. Ellis *et al*, Nucl. Phys. B373 (1992) 399; S. Sarkar, Oxford preprint OUTP-95-16P, hep-ph/9602260.
- [12] U. Ellwanger, Phys. Lett. B133 (1983) 187; J. Bagger and E. Poppitz, Phys. Rev. Lett. 71 (1993) 2380; J. Bagger, E. Poppitz and L. Randall, Nucl. Phys. B455 (1995) 59; V. Jain, Phys. Lett. B351 (1995) 481.

- [13] M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B159 (1979) 429; For reviews see S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, '*Superspace or 1001 Lessons in Supersymmetry*', (Benjamin/Cummings, 1983); P. P. Srivastava, '*Supersymmetry, Superfields and Supergravity*', (Adam Hilge, 1986).
- [14] see A. H. Chamseddine and H. K. Dreiner, Nucl. Phys. B458 (1996) 65 and references therein.